MEASURING DISAGREEMENT IN GROUPS FACING LIMITED CHOICE PROBLEMS[1]

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Abstract

A measure of the amount of disagreement, ***D****, in a group facing a problem with limited solution choices is proposed.* ***D*** *is simple to calculate, meaningfully derived and provides a standard scale from 0 to 1 for the disagreement of any size group facing a number of solution choices. It also provides a related measure,* ***d****, which allows the measurement of the disagreement of each individual in the group.* ***D*** *essentially compares the number of differences found between pairs of individuals in the group with the number of differences theoretically possible. Extension of the measure to the case where the solution choices are represented by ranked, interval and ratio scale data shows that* ***D*** *is equal to twice the variance of the solution scores, although in this case the maximum value of* ***D*** *may be greater than 1. The properties of this measure are explored and found to be similar to what is expected of a measure of disagreement. An example application is given, illustrating how disagreement at both the individual and group levels can be meaningfully and usefully represented by* ***d*** *and* ***D****.*

Introduction

In the study of groups, generating group agreement and decision acceptance can be as important as generating a decision solution [1, 2]. The measurement of agreement is therefore an increasingly important subject. A simple way to measure agreement is in terms of *commonality,* or the number of people who have the same idea [3, p364]. This measure however ignores the amount of disagreement among the remainder of the group, giving only the agreement for one solution option. Another method is to instruct the group to reach *consensus* or unanimity and to count the percentage of unanimous groups [4]. Such methods ignore the varying degrees of agreement possible in groups who achieve less than complete unanimity. Recent experiments with electronic groups have used a more sensitive measure of group agreement [5-7] derived from the mathematics of fuzzy set theory [8], and calculated by means of a computer program. However this method only works with interval data, not nominal or ordinal data, such as multi-choice questions [9]. This paper outlines an alternative, simpler measure, which can be applied to interval, ordinal and nominal data, and was used by the first author in an experiment with electronic groups facing multi-choice questions [10].

Definition

It was found to be more natural to develop a measure of *disagreement* rather than agreement, as has previously been done, although the two obviously relate. Disagreement was conceptualized in terms of the *square of the distance apart* of the positions ($\mathbf{R_i}$, $\mathbf{R_j}$) held by two group members. If two members hold the same position, their distance apart is zero, and their disagreement is zero. The situation under consideration is where **N** (N > 1) group members face a problem with **K** (K > 0) mutually exclusive response options (A, B, C, ...). The disagreement between any two group members is $\bullet_{ij} = (R_i - R_j)^2$

The disagreement ($\mathbf{d_i}$) between one person who chooses option **i** and the rest of the group can be defined as the sum of the disagreement between that person and each of the other group members, divided by the possible number of relationships:

[1] Many thanks to Ray Littler of Waikato University for pointing out the connection between our measure of disagreement and Simpson's measure of ecological diversity.

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$$d_i = \frac{1}{(N-1)} \sum_{1 \le j \le K} \bullet_{ij} f_j$$

$$= \frac{1}{(N-1)} \sum_{1 \le j \le K} (R_i - R_j)^2 \; f_j$$

where $\mathbf{f_j}$ is the number of people who chose option j. If all other participants choose the same option, then $\mathbf{d = 0}$ (no disagreement), whereas if everyone chooses a different option then $\mathbf{d = 1}$, maximum disagreement.

For nominal data, if two people choose different options their disagreement, then $\bullet_{ij} = 1$. In this case, if $\mathbf{N_i}$ group members choose option **i**, then the disagreement of one individual choosing option **i** is the number of disagreements they have with the rest of the group $(\mathbf{N - N_i})$ divided by the number possible disagreements $(\mathbf{N - 1})$, so $\mathbf{d_i} = \frac{N - N_i}{N - 1}$

Table 1 gives an example of how individual disagreement could be measured for a group of five members (N=5) given four solution choices (K=4), namely A, B, C and D. The disagreement of the individual is the number of others who disagree with them, divided by the maximum number of disagreements.

Individual disagreement	Individual response	Rest of group response
0.0	A	AAAA
0.25 (1/4)	A	AAAB
0.5 (2/4)	A	AACD
0.75 (3/4)	A	ABBC
1.0 (4/4)	A	BCCD
1.0 (4/4)	A	BBBB

Table 1. Individual disagreement values for N=5 subjects and K=4 options

The disagreement for the group (**D**) can be obtained by averaging the disagreement of its members:

$$D = \frac{1}{N} \sum_{1 \le i \le K} f_i \, d_i$$

$$= \frac{1}{N.(N-1)} \sum_{1 \le i \le K} \sum_{1 \le j \le K} (R_i - R_j)^2 \; f_j \, f_i$$

For nominal data this becomes

$$D = \frac{\sum_{1 \le i \le K} N_i \, d_i}{\sum_{1 \le i \le K} N_i} = \frac{N^2 - \sum N_i^2}{N^2 - N}$$

Where $\mathbf{N} = \sum_{1 \le i \le K} N_i$ and the minimum value of **D** is 0, when all members of the group agree.

Table 2 shows an example of possible group disagreement (**D**) values. As can be seen the maximum **D** value of 1.0 (everyone disagrees) is not possible here, because there are five group members but only four choices. The line indicates where the group moves from majority agreement to being unable to make a majority decision.

Group disagreement		Group response form	Example
0.0		Unanimous	AAAAA
0.4	(8/20)	All but one	AAAAB
0.6	(12/20)	3-2 split	AAABB
0.7	(14/20)	3-2 majority	AAABC
0.8	(16/20)	Hung group	AABBC
0.9	(18/20)	Maximum disagreement	AABCD

Table 2. Group disagreement measure for N=5 and K=4

This definition of **D** can be compared to the use of the index of the actual number of mutual friendships in a group divided by the number of possible mutual friendships as "one of the best indicators of a group's cohesion" [11, p123], although in this paper what is being considered is solution choice, not friendships formed. **D** measures how much disagreement occurs relative to how much is theorctically possible.

The maximum value of **D** is the maximum separation of subjects, which is not necessarily the same as the maximum polarization. For example a group response of AAABB would show maximum polarization (into two groups), while a response of AABCD shows maximum disagreement (of everyone).

The relationship between D and variance

For the numeric response case the value of **D** can be shown to be directly related to the variance of the responses:

$$D = \frac{1}{N.(N-1)} \sum_{1\le i\le K} \sum_{1\le j\le K} (R_i - R_j)^2 \; f_j f_i$$

$$= \frac{1}{N.(N-1)} \{ \sum_{1\le i\le K} \sum_{1\le j\le K} R_i{}^2 f_j f_i + \sum_{1\le i\le K} \sum_{1\le j\le K} R_j{}^2 f_j f_i - 2 \sum_{1\le i\le K} \sum_{1\le j\le K} R_i R_j f_i f_j \}$$

$$= \frac{1}{N.(N-1)} \{ 2N \sum_{1\le i\le K} R_i{}^2 f_i - 2 (R_i f_i)^2 \}$$

$$= \frac{1}{N.(N-1)} \; 2N (N\text{-}1) s^2 \quad = 2s^2$$

In other words the group disagreement is equal to twice the variance. This provides some basis for confidence in **D**, since variance seems a reasonable and reliable measure of disagreement for numeric data.

Maximum value of D

The maximum disagreement of 1.0 is only possible if the number of people in the group is less than or equal to the number of choices ($N \le K$), when it is possible for everyone to disagree. If the number of choices is less than the number of group members ($N > K$) it is not possible for everyone to disagree. In this case the maximum group disagreement **D** is less than 1.

The maximum value for **D** is attained when the group is spread as evenly as possible over all **K** options. Suppose **r** is the integral quotient and **a** is the remainder when N is divided by K, so that $N = rK + a$. Then **D** will be maximized when all the N_i values are as close to **r** as possible.

Thus let $N_i = r$ for $i = 1, 2, \ldots K - a$ and $N_i = r + 1$ for $i = K - a + 1, \ldots. K$. Then the maximum value of **D** will be:

$$D_{max} = \frac{N^2 - (K-a)r^2 - a(r+1)^2}{N^2 - N}$$

If $N \le K$ then obviously $D_{max} = 1$, however if $N > K$ then the maximum value of **D** is less than 1. This reflects the fact that if there are less choice options than people in the group some people must agree (select the same choice option). Table 3 shows how the maximum value of **D** reduces from 1.0, as the group size increases, for selected vales of **K**. In general, as **N** gets very large, $\mathbf{D}_{\mathbf{max}}$ tends towards **1 - 1/K**. For example in the case where there are two solution choices (**K = 2**), as **N** becomes very large, $\mathbf{D}_{\mathbf{max}}$ tends to 0.5. This suggests that large groups facing two choice problems can experience only half the disagreement that can occur for small groups facing the same choices.

	Maximum D			
Group size	**K = 2**	**K = 3**	**K = 5**	**K = 10**
2	1.000	1.000	1.000	1.000
3	0.667	1.000	1.000	1.000
5	0.600	0.800	1.000	1.000
10	0.556	0.733	0.889	1.000
100	0.505	0.673	0.808	0.909
1,000	0.501	0.667	0.801	0.901
1,000,000	0.500	0.667	0.800	0.900

Table 3. Maximum D by increasing N for various K

Discussion

The advantages **D** and **d** as measures of group and individual disagreement are:

1. **Simple. D** and **d** are simple enough to be calculated by hand for small groups.
2. **Sensitive.** As can be seen from Table 2, **D** recognizes that a group response of **AAABC** (**D = 0.7**) shows more disagreement than a group response of **AAABB** (**D = 0.6**).
3. **Valid. D** and **d** are derived from a definition of the disagreement between individuals which is meaningful in terms of what is normally understood to be disagreement.
4. **Scaled.** For nominal data **D** and **d** have a fixed scale, from 0 (unanimity) to 1 (everyone disagrees) regardless of group size, although the maximum value of **D** is less if there are more people than choices.
5. **Related.** There are advantages in group research having related measures of individual and group disagreement.

It is an interesting question whether the properties of **D** relate to what is known about group disagreement, and whether the measure itself has any implications in this area. For example the reduction of $\mathbf{D}_{\mathbf{max}}$ as group size increases, shown in Table 3, suggests that for a problem with a relatively low number of solution choices, increasing group size decreases the maximum disagreement possible in the group. Since **D** is the average of **d**, a possible benefit of larger decision groups is a reduction in the maximum individual and group disunity that can occur. While from a task result perspective, the optimal group size for face-to-face interactions may be as low as five or six [12], it may be that larger groups are better from a social, group agreement perspective, because they have less potential for disagreement, especially for problems with few choices. Larger groups can handle limited choice problems with less threat to group unity.

Table 1 shows that when a group of five moves from consensus to one person disagreeing, the dissenting individual's disagreement changes from **d = 0.0** to a maximum of **d = 1.0**. This seems to reflect the nature of the situation. To break unanimity an individual must disagree with all the other members of the group, and it is not possible for them to disagree with only one or two of the others. To effect such a major change can be expected to be difficult to do, as it involves a change from no disagreement to maximum disagreement. Table 2 shows that for the consensus group as a whole, when one person disagrees, the group **D** registers a change of almost half the scale (**D = 0.4**), suggesting that for a group of this size the disagreement of one person is a major event. This effect however reduces as group size increases. For a group of ten, one person disagreeing gives a **D** value of **0.2**, and for a group of 100 the **D** value is only **0.02**. Thus the larger the group, the less important one person disagreeing is, which seems reasonable. There seems to be an certain inherent stability in consensus, especially for small groups.

D is a function not only of the group size but also the number of solution choices. The number of solution choices can be taken to be a measure of the complexity of the problem. More complex problems provide more choices. For a given size group, the more choices confronting it the more disagreement can occur, up to the point where there are as many choices as there are members in the group. Equivocal problems [13] are problems which are ambiguous - it is not even clear what the problem is. Could such problems be taken to be the special case where the number of solution options is infinitely large? Such problems could generate maximum disagreement in any size group.

Ecological diversity

An equivalent formula to that derived here can be found in mathematical ecology, where a given habitat can have **N** creatures and **K** species. If all **N** creatures are of the same species (**K** = **1**) then the ecological diversity is low, whereas if every animal is a different species (**K** = **N**) then the ecological diversity is high. Simpson's measure of ecological diversity [14, p223] applies to the case of **N** animals of which $\mathbf{N_j}$ belong the **j'**th species (**j** = 1,2, ... **K**). The probability of choosing one animal without replacement from species **j** is ($\mathbf{N_j}$ / **N**), and the probability of choosing a second from the same species is ($\mathbf{N_j}$ - 1)/ (**N** - **1**). Summing the probability of choosing two animals of the same species over all species and subtracting from 1 gives Simpson's measure:

$$\mathbf{D = 1 - \sum_{1 \le i \le K} \frac{N_j(N_j - 1)}{N(N-1)}}$$

which may be rearranged to give

$$\mathbf{D = \frac{N^2 - \sum N_i^2}{N^2 - N}}$$

which is the same as the group disagreement measure defined earlier for the nominal case. It is interesting that two situations so different as group disagreement and ecological habitat diversity can give rise to the same formula, derived in different ways. The measure is the same, although the situations seem different. For example subjects asked to choose again may show a test-retest correlation less than one, while animals by contrast do not change their species on re-sampling. There may be a higher concept that incorporates both group disagreement and ecological diversity in the same theoretical framework, where the species of an animal equates to the choice of an individual.

Probability distribution

Assuming all solution options are equally likely to be chosen by all group members gives the probability distribution as shown in Table 4 for N = 5 and K = 4. The distribution of **D** is positively skewed, suggesting that there are more ways a group can disagree than there are ways they can agree. The mean value of **D** given the distribution shown is 0.75, which is half way between the smallest majority (a 3-2 majority) and a hung group, as represented by the line in Table 4. This line represents the point at which the group loses the ability to form a majority decision.

Group disagreement	Group response	Example	P( )
0.0	Unanimous	AAAAA	4/1024
0.4 (8/20)	All but one	AAAAB	60/1024
0.6 (12/20)	3-2 split	AAABB	120/1024
0.7 (14/20)	3-2 majority	AAABC	240/1024
0.8 (16/20)	Hung group	AABBC	360/1024
0.9 (18/20)	Maximum disagreement	AABCD	240/1024

Table 4. Probability distribution for D (N=5 and K=4)

A distribution such as shown in Table 4 can be used to define a null hypothesis that all solution options are equally likely. It represents a situation where subjects have no reason to select one option over another, either because the problem is very difficult, and subjects do not know which option is correct, or because subjects are given choices they find equally attractive. By contrast, for an easy problem such as 2 + 2 = ? the correct answer will have a high probability and the others very low ones. The same situation would arise when subjects had a strong preference or bias for one option.

This null hypothesis does not recognize the distinction between intellective problems (which have a right/wrong answer) and preference problems (whose answer depends on user preference), as defined in McGrath's task circumplex [15]. An even distribution of the probability of selecting solution options could occur either because subjects found an intellective problem very difficult, or because they had no particular bias on a preference problem. The key factor is whether all response options have equal probability. Indeed from a subject's point of view, there may be very little difference between a difficult intellective problem and a preference problem, where subjects choose the option they prefer. The *response probabilities* of the subjects can be used to define the choice situation, rather the experimenter defined task context.

Example application

This measure was used in an experiment where computer-mediated groups of five subjects, interacting only through a computer network, had to provide group solutions to multi-choice questions with four choice options [10]. The treatment group first chose their answer without seeing how others voted. On their second vote round however they could see how the group voted on the first vote, and likewise on the third vote round they could see the second round votes. The control group simply voted alone without knowing how the others voted. The treatment situation was designed to minimize personal and informational influence, while retaining the necessary conditions for normative influence. Subjects could not discuss the questions at all, and all responses were anonymous. The main dependant measure of the experiment was group disagreement (**D**), which showed a highly significant effect (F = 242.6, p < 0.000***). The disagreement measures made it possible to analyze the data in further detail. Before their second vote subjects could find themselves in the minority (**d** $\geq$ **0.75**) or in the majority (**d < 0.75**). It was expected that those in the minority would tend to change their vote and those in the majority would not.

The results, shown in Table 5, indicate a clear trend in the expected direction, with a threshold effect once the voter moves into a minority, as has been found in other studies [16]. Individuals in a minority seem to be influenced by the rest of the group. However individual disagreement (**d**) alone does indicate whether the rest of the group agreed or disagreed among themselves. For example if an individual disagrees with everyone else, then they are in a minority, but if the rest of the group disagrees with each other as well, then everyone is in the minority. In this case there is no majority to attract the individual to change position. It was proposed that an individual disagreeing with the rest of the group would be more likely to change their position if the rest of the group agreed among themselves, and a group disagreement score was calculated *for the other four members* of the group, called **DRest**. Table 6 shows the possible combinations of **d** and **Drest** and in each cell gives an example of the vote situation, e.g. AAAA**B** describes the case where the individual (shown in bold) finds that one person in the group disagrees with them. As can be seen from Table 6, not all combinations of **d** and **Drest** are possible.

Table 7 shows the percentage who changed their vote position in each situation, and the number of times that combination occurred (in brackets).

Vote 2	Individual disagreement (d)				
	0.0	**0.25**	**0.5**	**0.75**	**1.0**
% changed vote	0.8%	2.0%	7.1%	36.3%	72.6%
N	479	403	368	355	551

Table 5. Percentage of changed votes by prior individual disagreement

Individual disagreement (d)	Rest of group disagreement (Drest)				
	0.0 **AAAA**	**0.5** **AAAB**	**0.67** **AABB**	**0.83** **AABC**	**1.0** **ABCD**
0.0 Disagree with no-one	AAAAA				
0.25 Disagree with one		AAAAB			
0.5 Disagree with two			AAABB	AAABC	
0.75 Disagree with three		BAAAB		BAABC	BABCD
1.0 Disagree with all	DAAAA	DAAAB	DAABB	DAABC	

Table 6. Examples of individual by rest of group disagreement combinations

Individual disagreement (d)	Rest of group disagreement (Drest)				
	0.0 **AAAA**	**0.5** **AAAB**	**0.67** **AABB**	**0.83** **AABC**	**1.0** **ABCD**
0.0 Disagree with no-one	**1.0%** (1625)				
0.25 Disagree with one		**2.9%** (886)			
0.5 Disagree with two			**8.3%** (289)	**3.8%** (261)	
0.75 Disagree with three		**45.0%** (238)		**25.3%** (190)	**14.7%** (34)
1.0 Disagree with all	**66.1%** (369)	**68.4%** (247)	**77.0%** (61)	**60.3%** (78)	

Table 7. Vote change by vote situation

The results in Table 7 follow an interesting pattern. Firstly 83.6% of all vote events involved no change in vote position, so subjects tended to stay with their previous vote position. For **d** = **0.5** and **d** = **0.75** the percentage who changed position decreased as the others in the group disagreed more, as expected. However when **d** = **1.0** and the subject disagrees with everyone, a majority of four in agreement against the subject produces no greater effect than a majority of three, perhaps indicating some sort of threshold has been reached. The case **D**AABB is interesting in that it involves two other competing candidates for the group majority, and produced the highest vote change. If the individual is identified with the group, and if the group needs agreement, then *the likelihood of vote change can be taken as the probability that the individual's solution choice will form a majority, compared to the probability that another solution choice can form a group majority*. These ideas can be put as propositions:

1. *Individuals will tend to maintain their previously adopted position,*

2. *The likelihood of no change depends on the probability that the individual's current position will form a group majority, with a threshold effect occurring at the minority/majority boundary,*

3. *The likelihood of change to a particular alternative depends on the probability that the alternative will form a group majority, and,*

4. *A small amount of change will occur regardless.*

These propositions could form the basis of a computer simulation of normative group behavior, using parameter values suggested by the results given. For example the probability of remaining with the previous position regardless seems to be about 33%, the random change factor about 1%, the majority-minority threshold from 0% to about 45%, and the likelihood of change induced by alternate positions from 2% to about 20%, depending on group situation. Such a program could operate dynamically, and simulate not only computer-mediated normative effects [17, 18] but also the sequence effects that occur when individuals respond in any order [19]. The measures proposed can be used to summarize large amounts of data in a meaningful way, and may be useful in theoretical models.

Conclusions

Measures of group and individual disagreement, **D** and **d**, have been proposed which are easy to calculate, meaningfully derived and appear to have the properties desired. For numeric data **D** is a simple transformation of the response variance. **D** and **d** provide sensitive measures of disagreement in groups facing problems with a given number of solution alternatives. They are also meaningfully related, as the group disagreement **D** is the average of the group member disagreements, **d**. As this paper has presented only a cursory analysis of **D** and **d** there may be problems in their application. However at this stage they appear to be a useful ways of measuring an increasingly important product of group activity - the amount of disagreement in the group. The measurement of disagreement in groups of varying size, facing problems with varying numbers of solution choices, could be a rich area for future research.

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